Effects of Bending Stiffness in Tow and Salvage Cables

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The effects of the small amount of bending stiffness that a tow or salvage cable are known to possess are considered. The results show that there are significant differences in the states of stress of a cable without stiffness and a cable with a small amount of stiffness in the neighborhood of a concentrated loading such as that arising due to a fitting. Furthermore, the magnitude of these differences are independent of the amount of bending stiffness. It is the extent of the region of the cable over which these differences occur that diminishes as the bending stiffness decreases. From an engineering point of view, the most important difference for a cable with stiffness might be that the direction of the principal stress changes dramatically with a change in position within the neighborhood of a concentrated loading. This fact could be of considerable importance since the strength characteristics of cables are extremely direction dependent. Also, there are surfaces in the cable across which a compressive force must be transmitted for small amounts of bending stiffness. The paper discusses these facts in detail. Order of magnitude estimates of the most important effects are presented.

Introduction

PREDICTING the response of a cable during a tow or salvage operation has been the source of an extensive and still growing literature.[1] The reason for the size of the literature is twofold. On the one hand, the basic problem is complicated enough that a complete solution has remained, and promises to continue to remain beyond the grasp of either our analysts or our computers. On the other hand, there are any number of physical approximations that may be introduced in order to make the problem tractable. Predictions based on these approximations are usually checked either with experiments or with similar predictions that have their source in a different set of approximations.

The most commonly accepted approximate model of the cable is that of a nonlinear inextensible string. For this model the prediction of the steady-state response of the cable to a planar load system may be reduced to evaluating integrals for the cases in which the loading either is a function of position alone or is a function of angle of attack alone.[2] Thus, from a mathematical point of view this simple but important problem may be considered solved. From an engineering point of view, however, it is necessary to actually evaluate the integrals for various combinations of loading conditions. Reference 1 contains a rather extensive bibliography of the results of these efforts.

In the present paper we consider a generalization of this commonly accepted model and allow the string to possess a small, but finite, amount of bending stiffness. Real tow and salvage cables do possess some bending stiffness, albeit a small amount of it. It is to be expected that neglecting this small amount of bending stiffness will result in a small error provided the axial tension in the cable does not become small and provided we are not interested in the state of stress of the cable in the vicinity of a concentrated loading. Unfortunately, there are important problems involving tow or salvage cables in which one (or both) of thees two provisos is (are) not applicable. For example, cables frequently break near fittings which are sources of concentrated loadings.

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Thus, the critical state of stress is considerably affected by the bending stiffness, no matter how small. As another example, Pao[3] recently identified and investigated a flutter type instability that arises during some tow conditions. This instability which is associated with regions of axial compression are greatly affected by the bending stiffness of the cable. (We note that Pao's analysis which incorporated the ability of the cable to resist bending was based on a linear theory). Finally, as in any approximate analysis, it is essential that we be able to supply some quantitative measure of the error involved. We shall consider, here, only the first and third of these points. The instability identified by Pao is a dynamic instability and we leave the further investigation of it within the framework of a large deflection theory to later studies.

While the effects of the bending stiffness have not received much attention in the literature on tow or salvage cables, they have not been neglected in the literature on closely related problems. References 4–6 represent just a partial bibliography pertaining to the problem. In addition to applying the mathematical results of the literature to tow or salvage cables, our efforts also represent a contribution to this literature. The contribution is a formal asymptotic expansion that uniformly converges to the exact answer in the limit of vanishingly small bending stiffness. In the present report, however, we consider only the application of the mathematical end results to tow or salvage cables. The development of these end results is reported elsewhere.[7]

The outline of the report is as follows: the first section consists of a statement of the specific physical problem to be studied and the mathematical formulation to which this problem leads. The second section gives a first-order approximation of the solution of the mathematical problem together with analytic expressions for the errors to assign to this approximation. These errors vanish in the limit of zero bending stiffness. The physical implications of the first-order approximation are discussed. In particular, emphasis is placed on the differences between the response of a cable with small bending stiffness and that of a string (i.e., cable with zero bending stiffness). In section three, attention is directed to obtaining order of magnitude estimates of the effects of the bending stiffness for typical tow or salvage cables. In this section we also present some estimates of the error to ascribe to our analysis. In a final section the major conclusions reached are synopsized.

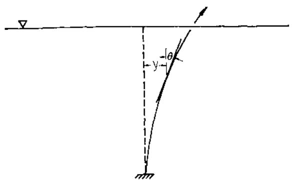

Fig. 1 Schematic representation of specific problem that is analyzed.

I. Description and Formulation of Problem

The geometry for the specific problem to be studied is shown in Fig. 1. The beam-string (i.e., a string with a small amount of bending stiffness) is taken to be fixed at the sea floor. That is, the point of contact with the floor and the direction of the axis of the beam-string at this point of contact are specified. At the surface a force is applied to the end of the cable. This force is known both in magnitude and direction. The manner in which the force is applied insures that there is no moment applied at this end. The beam-string is also taken to be subject to distributed forces acting along its length. The distributed forces and the end force all lie in the same plane so that the deflected shape will define a planar curve. This model is thought to be a valid representation of the loading on a cable during some salvage operations.

In order to obtain a mathematical problem that lends itself to an analytical solution, it would be necessary to restrict the distributed forces to those that depend on position alone or to those that depend on the angle of attack alone. The first case is thought to be an approximation of the loading that occurs in certain shallow water salvage operations in which the loading arises chiefly due to the weight of the cable and to drag forces resulting from a flowing current. The second case represents a better model for cables in deep waters.

The mathematical model employed is a result of the following assumptions which serve to make the problem tractable while still retaining the essential characteristics of the cable. 1) Elementary beam theory with no shear deformation governs the flexure of the beam-string. Large deflections will be allowed. 2) The beam-string is inextensible. The neglect of shear deformation in a long slender beam is valid. Inextensibility of the cable has long been accepted in tow cable studies.

Consistent with these approximations, the equations governing the response of the cable are taken in the following form

$$M' = -N \tag{1}$$

$$(T \sin\theta)' + (N \cos\theta)' = -f(\theta,s) \tag{2}$$

$$(T \cos\theta)' - (N \sin\theta)' = w(\theta,s) \tag{3}$$

$$EI\theta' = M \tag{4}$$

and

$$y' = \sin\theta \tag{5}$$

Here, M, N and T denote the bending moment, shear force, and tensile force, respectively, that are transmitted across a section of the cable; y denotes the horizontal deflection of the cable and θ denotes the angle made by the tangent to the cable and the vertical. These solution variables all vary with distance measured along the cable, which is given by s. Further, f and w are components of the force distributions in the horizontal and vertical directions. The forcing varies with θ and s. A prime denotes differentiation with respect to s and EI denotes the bending stiffness of the cable.

Equations (1–3) are obtained by requring equilibrium of an elemental length of the cable. Equation (4) is the load-deformation relationship of elementary beam theory. Equation (5) is obtained from geometry.

The governing equations are to be solved subject to the following boundary conditions which are simply mathematical statements of the end conditions already discussed. At the end $s = 0$, we require

$$y(0) = 0 \tag{6}$$

$$\theta(0) = 0 \tag{7}$$

whereas at the end $s = L$, we require

$$T \sin\theta + N \cos\theta = F \sin\gamma \tag{8}$$

$$T \cos\theta - N \sin\theta = F \cos\gamma \tag{9}$$

and

$$M = 0 \tag{10}$$

Here, F denotes the magnitude of the end force and γ gives the angle that this force makes with the horizontal. Also, L denotes the length of the cable.

Equations (1–10) define a well-posed boundary value problem on the five unknowns; T,N,M,θ and y. We note that Eqs. (1–4) and (7–10) define a well posed problem on the four unknowns; $T,N,M,$ and θ. Once this problem is solved, y is given by a simple integral defined by Eqs. (5) and (6). In our studies we are primarily interested in the internal forces and moments and will have only a peripheral interest in the deformed shape. We will, therefore, simply ignore the integration needed to obtain y, once θ has been determined.

In the next section, we give a first-order approximation of the solution that is valid for small EI and a discussion of the error to associate with this approximation. The actual construction of the first-order approximation is accomplished in Ref. 7.

II. First-Order Approximation and Error Estimates

In Ref. 7 is constructed an asymptotic series that converges to the solution of the mathematical problem posed in the limit of vanishingly small bending stiffness. The construction procedure used is usually termed the method of "matched asymptotic expansions." No details of the construction are reproduced here. The results are written:

$$\theta(s) = \theta_s(s) - \theta_b(s) + \epsilon^{1/2}\theta_1(s) + \epsilon\theta_2(s) + 0(\epsilon^{3/2}) \tag{11}$$

$$T(s) = T_s(s) - T_s(0)[1 - \cos\theta_b(s)] + \epsilon^{1/2}T_1(s) + \epsilon T_2(s) + 0(\epsilon^{3/2}) \tag{12}$$

$$N(s) = T_s(0)[\sin\theta_0(s)] + \epsilon^{1/2}N_1(s) + \epsilon N_2(s) + 0(\epsilon^{3/2}) \tag{13}$$

$$M(s) = \epsilon^{1/2}M_1(s) + \epsilon M_2(s) + 0(\epsilon^{3/2}) \tag{14}$$

when

$$\theta_b(s) = 4 \arctan\{\tan[\theta_s(0)/4] \exp(-s/l)\} \tag{15}$$

$$\epsilon = EI/T_c l_c^2 \tag{16}$$

$$l = (EI/T_s(0))^{1/2} = (\epsilon T_c/T_s(0))^{1/2} l_c \tag{17}$$

The following words of explanation are necessary for an understanding of these equations. The bending stiffness enters the expressions defining the response via the non-dimensional parameter, ϵ, and the length measure l. In their definitions, T_c denotes some measure of the average tension in the cable over a segment of length l_c. For many problems of interest, the orders of magnitude of the end tension, F, and the load distributions, $f(s)$ and $w(s)$ are such that the distribution of tension along the cable, $T(s)$, does not vary too much from F. In such a case, we can treat the entire cable as one segment in which case $T_c = F$ and $l_c = L$. We note that a cable without bending stiffness (i.e., a string) corresponds to $\epsilon \equiv l \equiv 0$. For a typical tow cable

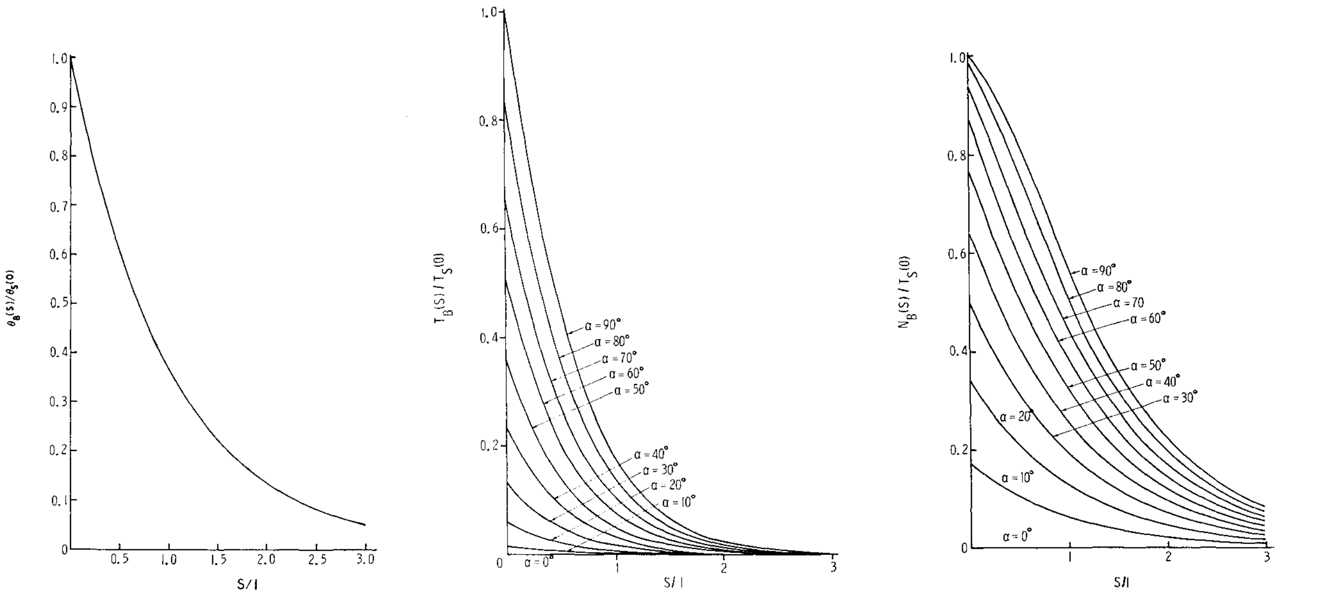

Fig. 2 Effect of bending stiffness near fixed end. a) Change of angle of attack with nondimensional distance. b) Effect of bending stiffness on tension as a function of nondimensional distance. c) Effect of bending stiffness on shear force as a function of nondimensional distance. For 1 in. cable, loaded to 40% of its ultimate strength, l is of the order of 7 in.

(e.g., a 1 in. cable 1000 ft long with $F = T_s(0)$ equal to 40% of the ultimate strength) ϵ is of the order of 10^{-7} and l is of the order of 7 in.

In Eqs. (11–14) θ_s and T_s define the response of a cable without any bending stiffness. The remaining terms all represent corrections that are to be applied for a cable with bending stiffness. They are presented in such a manner as to emphasize the dependence of the magnitude of a correction term on the amount of bending stiffness. Thus, for example, the magnitude of θ_b is independent of the amount of bending stiffness (i.e., given by ϵ), whereas $\epsilon^{1/2}\theta_1$ varies as $\epsilon^{1/2}$, and $\epsilon\theta_2$ varies as ϵ, etc. The notation $O(\epsilon^{3/2})$ denotes a term that has the same order of magnitude as does $\epsilon^{3/2}$.

From an engineering or physical point of view the most interesting feature of these expressions is that there are correction terms to be applied to θ, T, and N, that have magnitudes that are independent of the bending stiffness. The physical extent of the cable over which these correction terms are significant does depend on the bending stiffness, however. We note, for example, that $\theta_b(s)$ decreases from its maximum value at $s = 0$ to $1/e$ (where e is the Naperian constant) of this maximum in a distance equal to l. As noted previously, l depends on the bending stiffness of the cable. The correction terms to be applied to T and N show a corresponding decrease as s changes for 0 to l. Nondimensional plots of the variations of the correction terms discussed in this paragraph as a function of distance from the end at $s = 0$ are exhibited in Fig. 2.

We do not give the analytic expressions for the correction terms that have magnitudes that do change if the bending stiffness changes. In Ref. 7, boundary value problems are presented that are to be satisfied by these correction terms. Since these problems require the solution of linear differential equations with variable coefficients a general solution procedure is not available. Each case must be treated separately and this can only be done after the detailed variations of the force distributions $f(s)$ and $w(s)$ have been given. The smallness of the magnitudes of the corrections that these terms represent for most tow or salvage cables prevent their having much engineering significance.

These mathematical results lead to the following physical conclusions. Assuming that the tension in the cable does not approach zero anywhere along its length, then the cable with a small amount of bending stiffness will behave essentially as a string except over a distance of order of magnitude l measured from the point of end fixity. In this region the solution variables change rapidly from the values that they would have if the cable was indeed a string to the values they must satisfy by virtue of the end fixity conditions. Thus: 1) $\theta(s)$ changes from a value $\theta = \theta_s(0)$ at $s \approx l$ to a value $\theta = 0$ at $s = 0$. 2) $T(s)$ and $N(s)$ change from values $T = T_s(0)$ and $N = 0$ at $s \approx l$ to values $T = T_s(0)\cos\theta_s(0)$ and $N = T_s(0)\sin\theta_s(0)$ at $s = 0$, respectively. These last changes are necessary to maintain force equilibrium of the segment of the cable from $s = 0$ to $s \approx l$. 3) Equilibrium also requires that $M(s)$ change from a value of zero at $s \approx l$ to a value of $M = 2T_s(0)l\sin[\theta_s(0)/2]$ at $s = 0$.[7] Again, we emphasize that the magnitudes of changes in $\theta(s)$, $T(s)$ and $N(s)$ that arise due to some bending stiffness are independent of the amount of this bending stiffness. Only the length of the portion of the cable over which these changes occur varies with the amount of bending stiffness. In the limit of the bending stiffness approaching zero, the distance over which the changes in $\theta(s)$, $T(s)$, and $N(s)$ occur likewise approaches zero. The change that occurs in $M(s)$ does depend on the amount of bending stiffness and this change approaches a zero value as the bending stiffness vanishes. For tow and salvage cables the magnitude of $M(s)$ is small and the bending moment is of little engineering significance.

A mathematician would state the preceding conclusion by saying that the response of a beam-string does not approach that of a string uniformly over its length as the bending stiffness approaches zero. Rather for any finite bending stiffness, no matter how small there will be small regions over which the two responses are significantly different. As the bending stiffness is allowed to decrease, the extent of these regions of local differences decreases until in the limit the region has zero extent. This is a characteristic behavior of the type problem that an applied mathematician terms a singular perturbation problem. There are several well-known examples of such problems which may serve as analogues to the beam string problem. The flow of a fluid which possesses a small amount of viscosity provides the most direct analogy. A good description of the flow can be obtained by neglecting the viscosity except within certain limited regions. One such region is that which immediately surrounds an obstacle to the fluid flow. The boundary layer that arises due to the presence of some viscosity, no matter how small, is an exact analogue to our region of rapid change in $\theta(s)$, $T(s)$, and $N(s)$.

It is natural for an engineer with an interest in stress analysis of solids to attempt to draw another analogy and that is between the present phenomenon and that of a stress concentration. Some care should be exercised in drawing this latter analogy, however, since the mechanisms behind the two phenomena are essentially different, as are the end results of these mechanisms. We do not really have a concentration of stress in the present problem but really have a situation in which the orientation of the stress tensor changes significantly. We shall pursue this point further.

Just how important is the cable boundary layer to an engineer? The answer to this, of course, depends on what the engineer is predicting. If his primary interest is the deformed shape, the boundary layer is of little importance, since the shape of the cable is obtained by integrating $\theta(s)$. On the other hand, if he is worried about the possibility of failure of the cable, then it could be of primary importance since the possibility of failure will depend, among other things, on the tensile force and the shear force that must be transmitted across each section. (We are assuming here that the bending stiffness is so small that we need not consider the bending moment that exists in the boundary layer.) In the boundary layer we have a rapid increase in the shear force that is to be transmitted across the section at the same time that we have a rapid decrease in the tensile force that is to be transmitted. The net effect of the two opposing changes (opposing in the sense that one is detrimental from a possibility of failure point of view while one is beneficial) is difficult to assess quantitatively. Several qualitative statements can be made, however, and these statements may of greatest importance from a designer's point of view. First of all, we might note that the transmission of a tensile force across a section occurs more or less uniformly at each point of the section. On the other hand, the job of transmitting a shear force across a section falls more on the central portion of the section than it does near the edges. For example, if the cable behaves like a thin homogeneous bar, we might expect a distribution of normal stress that transmits a given tensile force and a distribution of shearing stress that transmits a given shear force to look something like that shown in Fig. 3. These sketches are meant to present only a schematic picture. The uneven manner in which the shear force is distributed serves to multiply the detrimental effect of the increase in shear force.

Secondly, we might consider Fig. 4 which shows, again schematically, the state of stress at a point on the section and the Mohr circle to which it gives rise. In the boundary layer we have a rapid increase in τ and a rapid decrease in σ. While we must know something of the rates of these changes vis-a-vis one another in order to predict whether the maximum tensile stress increases or decreases, there are two conclusions that can be drawn without such information. The first is that there will be surfaces of the cable across which a compressive force is to be transmitted as a result of the appearance of shear. The second is that the direction of the surface over which the maximum tensile stress is to be transmitted changes as a result of the appearance of shear. With respect to this first conclusion, we note that it is drawn independent of any Poisson's ratio type effect which is not included in the schematic. With respect to the second conclusion, we note that although the direction of the principal stresses is not very important for a body that is isotropic as far as its strength characteristics are concerned, a cable is not such a body. The strength characteristics of a stranded cable are extremely directional dependent, which makes the fact that the directions of the principal stresses change rapidly in the boundary layer a very detrimental occurrence.

Since the preceding discussion points to the fact that the occurence of a boundary layer is indeed detrimental, the obvious questions are "How do we eliminate it?" or "How do we live with it?" An obvious answer to the former question is to eliminate the end fixity condition. Introducing a fitting that is free to rotate, and thereby making the condition at the sea floor, in our problem, the same as that at the sea surface, serves to completely eliminate the boundary layer. Barring this possibility one might try to either introduce a stiffening element to the end of the cable or possibly try to go in the other direction and make the end of the cable less stiff. The conclusions that we have drawn point out the fallacy of this practice since the amount of stiffness does not affect what happens in the boundary layer but only affects the size of the boundary layer. (This last statement assumes that the amount of stiffness added is not so large as to destroy the analysis.) Another solution might be to learn to live with the boundary layer by reinforcing the end of the cable in such a way that its strength characteristics would not depend on orientation. This would eliminate what appears to be the most serious detriment of the boundary layer; namely, a sweeping of the angles in which the principal stresses act over a range of values.

As a further remark, we should consider the bending moment that occurs in the boundary layer. While the bending moment does decrease with a decrease in the bending stiffness, it does not decrease as fast as the errors that extend over the entire length of the cable. The latter errors decrease as ϵ, while the bending moment in the boundary layer decreases as $\epsilon^{1/2}$. Thus, there is a range of values of ϵ for which we are justified in neglecting the effects of the bending stiffness over most of the cable but must consider the bending moment that exists in the boundary layer. There is little more that one can say in this regard in addition to simply pointing out the possibility.

In the next section we shall try to give some numbers to give an order of magnitude estimate to the effects of the bending stiffness in a tow or salvage cable. These estimates should only be looked upon as describing the order of magnitude of the effects since the input numbers and several of the formulae used can only be considered accurate in an order of magnitude sense when they are applied to wire cables.

III. Some Numerical Results

In this section we should like to give at least order of magnitude estimates of the effects of bending stiffness in tow

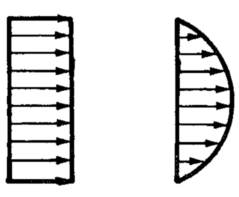

Fig. 3 Schematic representation of the distributions of the normal stresses and shearing stresses that arise to transmit a given normal force and shear force, respectively.

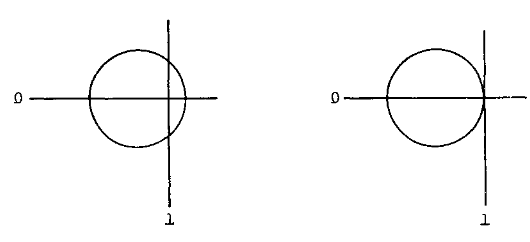

Fig. 4 Schematic representation of the Mohr circle that describe the state of stress in a cable. a) Mohr circle outside of boundary layer. b) Mohr circle inside of boundary layer.

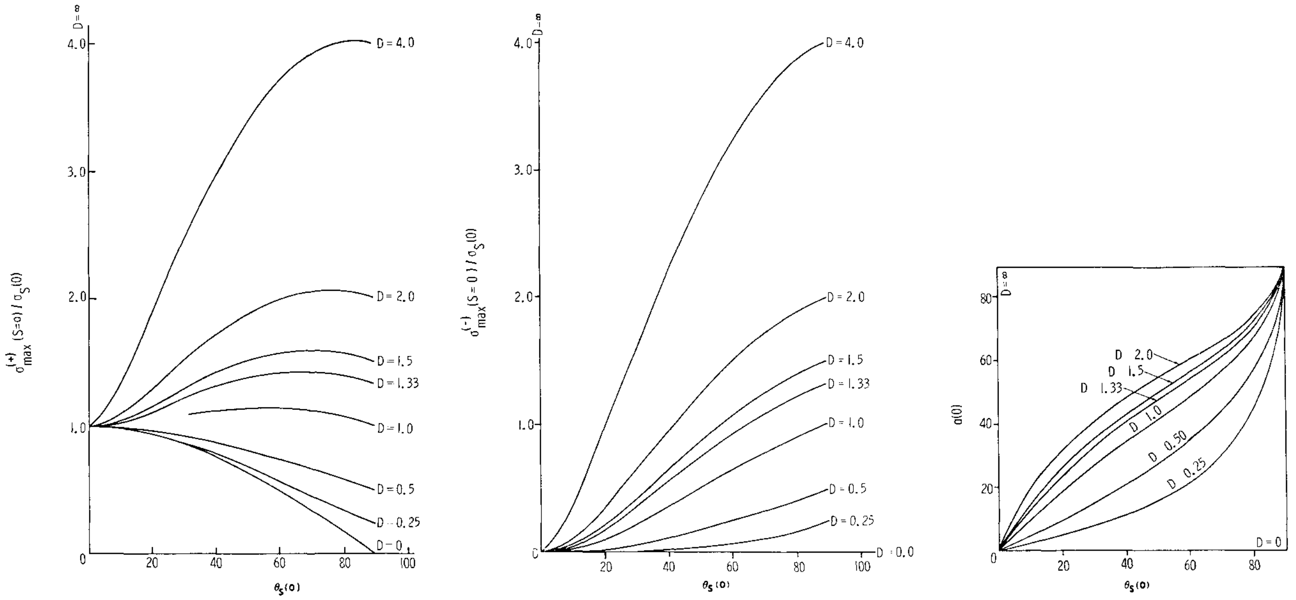

Fig. 5 Changes in magnitudes and directions of principal stresses as a result of bending stiffness in cables. $\theta_s(O)$ is kink angle at point of fixity, D is a geometrical property of cross section. a) Maximum tensile stress. b) Maximum compressive stress. c) Angle made by direction to maximum tensile stress and axis of cable.

or salvage situations. Specifically, we shall first of all give estimates of the changes in both the magnitudes and the directions of the principal stresses that occur in the boundary layer and secondly, we shall look to the size of the correction terms that are usually ignored.

In order to evaluate the changes in principal stresses that occur in the boundary layer it is necessary to determine the stress distribution that results from the requirement of transmitting a given shear force and a given tensile force. We ignore the bending moment to be transmitted across a section since the magnitude of this bending moment is small. (The bending moment is small because the shear force acts only over a small distance.) It is assumed that the cable behaves as a simple beam-column and draw on familiar strength of material formulas to give this distribution. We note that this assumption is crude at best and most assuredly limits the validity of our conclusions to situations in which the strands that make up the cable all act in unison. The appropriate formulae are

$$\sigma = T/A \tag{18}$$

$$\tau = NQ/It \tag{19}$$

In these expressions, A and I are the area and second moment of the section, respectively, Q is the first moment of that segment of the section that lies between the line along which we are calculating the shear stress and a free surface and t is the width of the section measured over the line along which we are calculating the shear stress. Thus σ is uniformly distributed over the section whereas τ is not. We shall be interested in τ_{max} which is given by

$$\tau_{max} = (N/I)(Q/t)_{max} \tag{20}$$

A simple Mohr's circle analysis gives the following expressions for the principal (or maximum normal stresses).

$$\sigma_{max}{}^{(\pm)} = (\tfrac{1}{2})[\sigma^2 \pm (\sigma^2 + 4\tau^2)^{1/2}] \tag{21}$$

where $\sigma_{max}{}^{(+)}$ is maximum tensile stress that occurs and $\sigma_{max}{}^{(-)}$ is maximum compressive stress. These stresses act on surfaces with outward normals that makes angles of α and $\alpha + 90°$ with the horizontal, respectively. Here, α satisfies

$$\sin\alpha = 2\tau/[\sigma + (\sigma^2 + 4\tau^2)^{1/2}] \tag{22}$$

Formulas given by Eqs. (20–22) are obtained from any elementary strength of materials text.

We are interested in evaluating the magnitudes of the two principal stresses and the angle defining the surfaces on which they act for the two sections located by $s = 0$ and $s \approx l$. These sections define the limits of the portion of the cable in which the effects of a small amount of bending stiffness are important. The state of stress at $s \approx l$ is that obtained if bending stiffness is ignored. It is the section at $s = 0$ which the error in ignoring the bending stiffness is greatest.

At $s \approx l$,

$$T(s \approx l) = T_s(0) \tag{23}$$

and

$$N(s \approx l) = 0 \tag{24}$$

This leads to

$$\sigma_{max}{}^{(+)}(s \approx l) = \sigma_s(0) = T_s(0)/A \tag{25}$$

$$\sigma_{max}{}^{(-)}(s \approx l) = 0 \tag{26}$$

and

$$\alpha(s = l) = 0 \tag{27}$$

At $s = 0$,

$$T(0) = T, (0) \cos[\theta_s(0)] \tag{28}$$

and

$$N(0) = T_s(0) \sin[\theta_s(0)] \tag{29}$$

This leads to

$$\sigma_{max}{}^{\pm}(0) = \sigma_{max}{}^{(\pm)}(s \approx l)\{\cos[\theta_s(0)](\cos\beta \pm 1)/2 \cos\beta\} \tag{30}$$

and

$$\sin[\alpha(0)] = \sin\beta/(1 + \cos\beta) \tag{31}$$

where

$$\beta = \arctan[2D \tan\theta_s(0)] \tag{32}$$

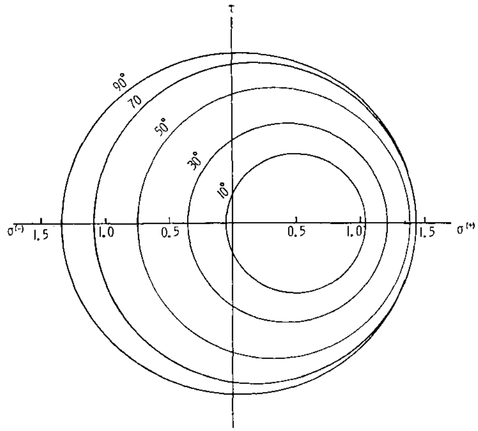

Fig. 6 Changes in Mohr circle of maximum stress state that arises due to presence of bending stiffness. θ_s (O) is kink angle at point of end fixity. Specific case of $D = \frac{4}{3}$, i.e., circular cross section, is exhibited.

and

$$D = (A/I)(Q/t)_{\max} \tag{33}$$

From Eq. (33) we see that D is a geometrical property of the section. For a circular section, $D = \frac{4}{3}$; for a rectangular section, $D = \frac{3}{2}$.

Plots of $\sigma_{\max}{}^{(+)}(0)/\sigma_s(0)$, $\sigma_{\max}{}^{(-)}(0)/\sigma_s(0)$, and $\alpha(0)$ are exhibited in Fig. 5, all as functions of kink angle. Kink angle is defined as the angle made by the slope of a cable without stiffness and the slope of a cable with some stiffness at the point $s = 0$. That is, $\theta_s(0)$. In the figures, D serves as a parameter. If we concentrate on the case for which $D = \frac{4}{3}$, i.e., circular cross section, we note that the maximum tensile stress is greatest for a kink angle of approximately 70° and that this greatest maximum tensile stress is approximately 50% greater than that which occurs in a cable without stiffness. The maximum compressive stress is always greatest for a kink angle of 90°. For $D = \frac{4}{3}$, the greatest maximum compressive stress is approximately 30% greater than the maximum tensile stress that occurs in a cable without stiffness. In Fig. 6 are plotted the changes in the Mohr circle for the maximum state of stress that results due to some cable stiffness. Specifically, the changes are exhibited as a function of kink angle for the case $D = \frac{4}{3}$.

Only a very rough estimate can be easily obtained for the error to ascribe to the results presented. We do this by simply stating that in the vicinity of the boundary, the error will be of the order of $\epsilon^{1/2}$, whereas away from the boundary the error will be of the order of ϵ. For a 1 in. cable, 1000 ft long, subject to a minimum tension force equal to 10% of its ultimate strength, ϵ is of the order of 10^{-6}.

IV. Conclusions

We synopsize the conclusions reached in this paper. 1) In an engineering analysis one is justified in treating a tow and salvage cable as a string over much of its length. In the vicinity of a concentrated loading such as that due to a fitting, however, the presence of even a small amount of bending stiffness will significantly affect the state of stress.

2) While one can expect some changes in the magnitudes of the principal stresses due to some bending stiffness, the most important changes occur in the orientations of the surfaces over which the principal stresses act. The orientation of surfaces is important for tow and salvage cables since the strength properties of a stranded cable are extremely direction dependent.

3) The amount of bending stiffness that a cable possesses determines the size of the neighborhood of a concentrated loading in which the changes that are due to the presence of this bending stiffness are important. The magnitudes of the changes that occur are independent of the amount of bending stiffness. Speaking qualitatively, decreasing the bending stiffness of a cable will result in a tighter kink in the vicinity of a concentrated loading. However, the principal stress that occurs in the kink does not change with a decrease in bending stiffness.

References

[1] Casarella, M. and Parsons, M., "Cable Systems Under Hydrodynamic Loading," *Marine Technology Journal*, Vol. 4, No. 4, July–Aug. 1970, pp. 27–44.

[2] Glauert, H., "Heavy Flexible Cable for Towing a Heavy Load Below an Aeroplane," Repts. and Memo 1592, Feb. 1934, Aeronautical Research Committee of Great Britain.

[3] Pao, H. P., "Dynamic Stability of a Towed Thin Flexible Cylinder," Institute of Ocean Science and Engineering Rept. 69-11, Themis Program N00014-68-A-0506, Dec. 1969, The Catholic University of America, Washington, D.C.

[4] Frisch-Fay, R., *Flexible Bars*, Butterworth, London, England, 1962.

[5] Plunkett, R., "Static Bending Stresses in Catenaries and Drill Strings," *Journal of Engineering for Industry*, Vol. 89, No. 1, Feb. 1967, pp. 31–36.

[6] DeRuntz, J. A., Jr., "End Effect Bending Stresses in Cables," *Journal of Applied Mechanics*, Vol. 36, No. 4, Dec. 1969, pp. 750–56.

[7] McCoy, J. J., "Effects of Bending Stiffness in Tow and Salvage Cables," Institute of Ocean Science and Engineering Rept. 71-2, Themis Program N00014-68-A-0506, Jan. 1971, The Catholic University of America, Washington, D.C.